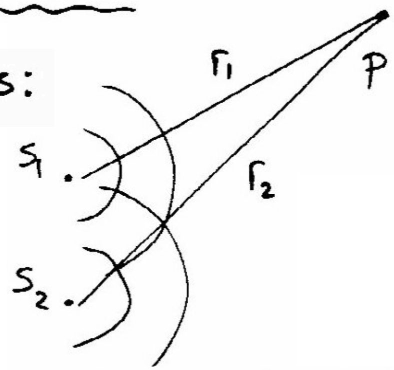
Chap 9 - Interference

eg 2 point sources:



at far point P:

for spherical waves, (same polarization)

$E_1 = E_{01}(r)\cos(kr - \omega t + \varepsilon_1)$ ← eval. at $r = r_1$

$E_2 = E_{02}(r)\cos(kr - \omega t + \varepsilon_2)$ ← eval. at $r = r_2$

$\alpha_1 \equiv kr_1 + \varepsilon_1$, $\alpha_2 \equiv kr_2 + \varepsilon_2$

then

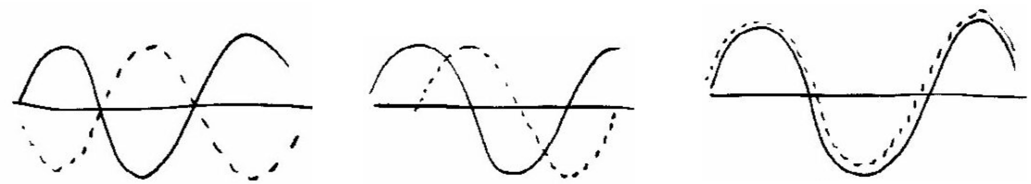
$$E = E_1 + E_2 = E_0 \cos(\omega t + \alpha)$$

with.

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$

and $\tan\alpha = \dfrac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2}$

composite wave is harmonic, same freq, but diff amplitude (and phase).



out-of-phase   in-phase

out-of-phase		in-phase
$\delta = \alpha_2 - \alpha_1 = \pi$	$\delta = \pi/2$	$\delta = 0$
$E_0^2 = (E_{01} - E_{02})^2$	$E_0^2 = E_{01}^2 + E_{02}^2$	$E_0^2 = (E_{01} + E_{02})^2$
$\lvert E_0 \rvert = \lvert E_{01} - E_{02} \rvert$		$E_0 = E_{01} + E_{02}$

recall $I \propto \langle E \rangle^2$; the $2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$ "interference term" is important !

examine $\delta = k(r_2 - r_1) + (\varepsilon_2 - \varepsilon_1)$

nonzero δ may arise from diff initial phases or from diff distances r ;

say, $\varepsilon_1 = \varepsilon_2$ and $E_{01}(r_1) \approx E_{02}(r_2)$,

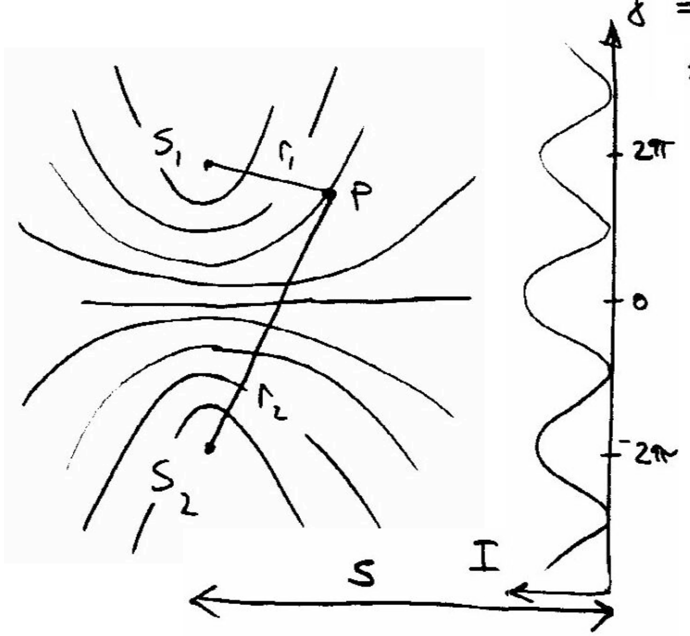
$$E_0^2 \approx 2E_{01}^2\left(1+\cos k(r_2-r_1)\right)$$
$$= 4E_{01}^2 \cos^2 \frac{k}{2}(r_2-r_1)$$
$$E_{01}^2 \equiv I_0$$

$$I = 4I_0 \cos^2\left[\tfrac{1}{2}k(r_2-r_1)\right]$$

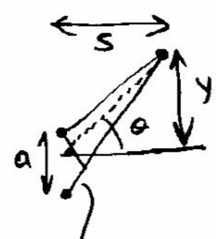
const. int. $r_2-r_1 = m\lambda \Rightarrow \frac{1}{2}k(r_2-r_1) = \frac{1}{2}km\lambda = \pi m$

dest. int. $r_2-r_1 = (m+\frac{1}{2})\lambda \Rightarrow \frac{1}{2}k(r_2-r_1) = \pi(m+\frac{1}{2})$

- get hyperboloids:



$$\delta = k(r_1-r_2)$$
$$\approx ka\sin\theta$$
$$= k\frac{a}{S}y$$



$r_1 - r_2 \approx a\sin\theta$

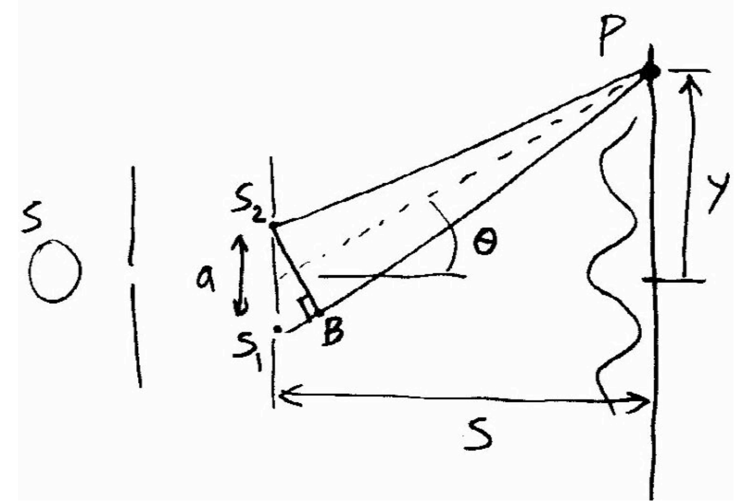
$y = S\cdot\tan\theta$
$\approx S\cdot\sin\theta$

(small θ)

- case of $E_{01}(r_1) \neq E_{02}(r_2)$ can be easily handled (ie. don't get completely zero intensity in sum)

- note: spatially average $I = 2I_0$ ✓ (intensity is redistributed)

thus, 2-slit setup:



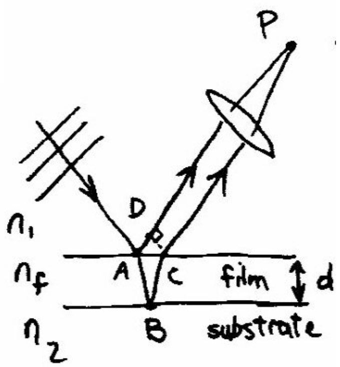
$\delta = ka\sin\theta = 2\pi m$ const. int.

or $\sin\theta_m \approx \theta_m = \dfrac{2\pi m}{ka} = \dfrac{\lambda m}{a}$

$$I = 4I_0\cos^2\left(\tfrac{1}{2}ka\sin\theta\right)$$

Amplitude-Splitting Interferometers:

2 beam interference in dielectric film:

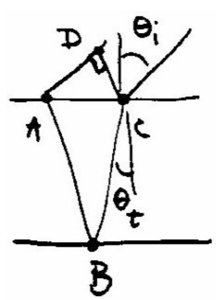


consider interference:

$$\Lambda = n_f\left(\overline{AB} + \overline{BC}\right) - n_1 \overline{AD}$$

but $\overline{AB} = \overline{BC} = d/\cos\theta_t$

$$\Rightarrow \Lambda = \frac{2n_f d}{\cos\theta_t} - n_1 \overline{AD}$$



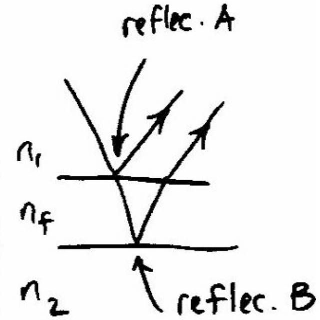
and $\overline{AD} = \overline{AC}\sin\theta_i = \overline{AC}\,\frac{n_f}{n_1}\sin\theta_t$

also, $\overline{AC} = 2d\tan\theta_t$

$$\Rightarrow \Lambda = \frac{2n_f d}{\cos\theta_t}\left(1 - \sin^2\theta_t\right) = \underline{\underline{2n_f d\cos\theta_t}}$$

phase difference: $\delta = k_0 \Lambda$ + phase shift due to reflections

vacuum value



for $\theta_i, \theta_t \lesssim 30^\circ$,

$\delta_A, \delta_B = 0$ or π

(depending if int. or ext. and on polarization)

$$\Rightarrow \delta_B - \delta_A = \pm\pi \text{ or } 0$$

$$\Rightarrow \delta = \frac{2\pi}{\lambda_0} 2n_f d\cos\theta_t + \left(\pm\pi \text{ or } 0\right)$$

if $\pm\pi$: choose $-\pi$,

for maxima: $\delta = 2\pi m = \frac{4\pi}{\lambda_f} d\cos\theta_t - \pi$

$$\Rightarrow d\cos\theta_t = \frac{\lambda_f}{4}(2m+1)$$

and minima at: $\delta = (2m \pm 1)\pi$

$$\Rightarrow d\cos\theta_t = \frac{\lambda_f}{4}(2m)$$

if 0:

maxima at: $d\cos\theta_t = \frac{\lambda_f}{4}(2m)$

minima at: $d\cos\theta_t = \frac{\lambda_f}{4}(2m+1)$

Example: $n_1 = n_2$

(film surrounded by same medium)

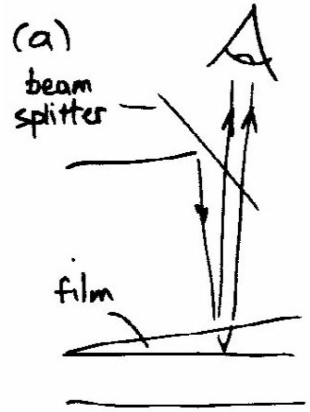
then reflec. A int and reflec. B ext or vice versa.

$$\Rightarrow \quad \delta_B - \delta_A = \pm \pi$$

Fringes of Equal Thickness:

vary $n_f d$ (or just d):

(a)

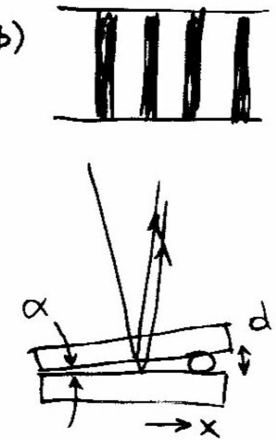


near normal incidence:

at each maxima,

$$d_m = \frac{\lambda_f}{2}\left(m + \frac{1}{2}\right)$$

(b)



Fizeau fringes

angle α

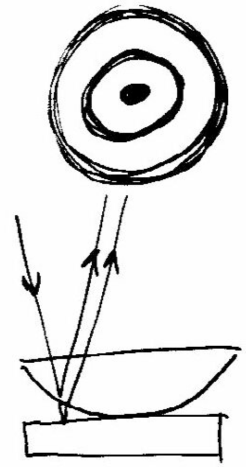
$(\delta_R = \pm \pi)$

$$d = x\alpha$$

$$= \frac{\lambda_f}{4}(2m+1)$$

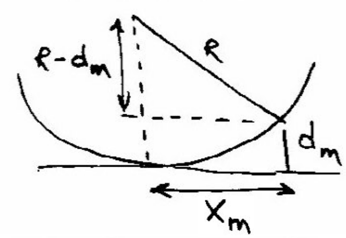
$$\Rightarrow \quad x_m = \frac{\lambda_f}{2\alpha}\left(m + \frac{1}{2}\right) \quad \text{or} \quad \Delta x = \lambda_f / 2\alpha$$

c)



Newton's rings

measure radius R:

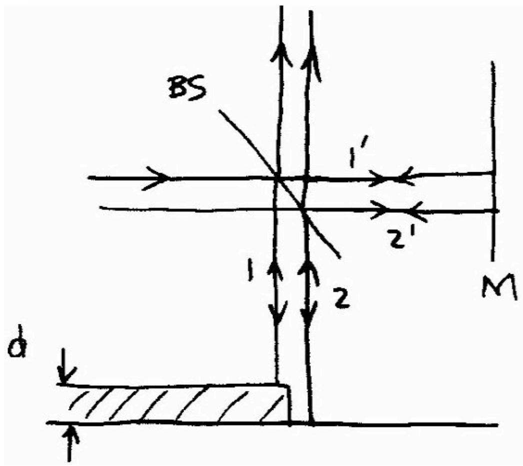


$$R^2 = x_m^2 + (R - d_m)^2$$

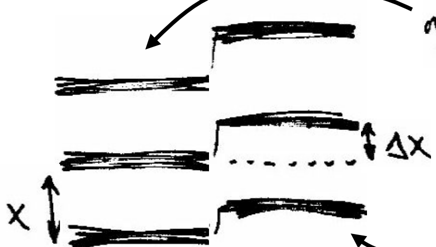
$$x_m^2 = 2Rd_m - d_m^2$$

$$\approx 2Rd_m$$

film thickness measurement:



- adjust position of M such that OPD between beams 1 and 1' is nearly const.
- with slight tilt of M, see:



max: $\frac{2\pi}{\lambda_0} 2t + \delta_R = 2\pi m$

t = OPD 1 and 1'

$t \rightarrow (t+d)$

have $\frac{2\pi}{\lambda_0} 2(t+d) + \delta_R = 2\pi (m' + \Delta m)$

take $m' = m$, $\frac{2}{\lambda_0} d = \Delta m$, but $\Delta m = \Delta x / x$

$$\Rightarrow \boxed{d = \frac{\lambda_0}{2} \cdot \frac{\Delta x}{x}}$$

notes:

1) in general will have phase shift due to reflec. from metal, but that's the same for 1, 1' and 2, 2'

2) how to determine if $m = m'$?

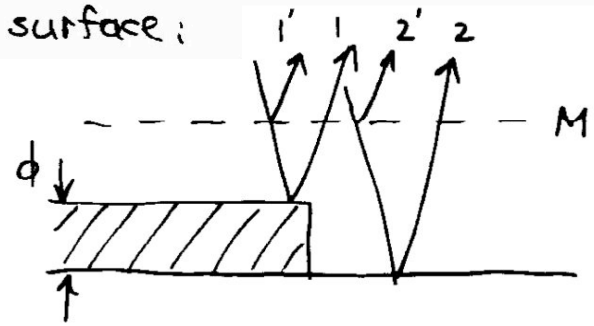
   - grade the height transition
   - use white light source, and look for $m=0$ or $m'=0$

   $m=+1$ } dispersed (rainbow)

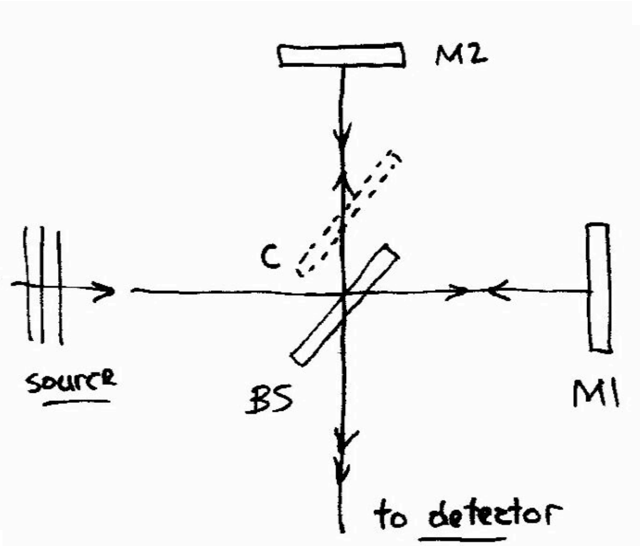
   $m=0$ } white

   $m=-1$ } dispersed

3) in analysis, replace M by virtual source at M', and interference arises from interference due to "air film" between M' and actual surface:



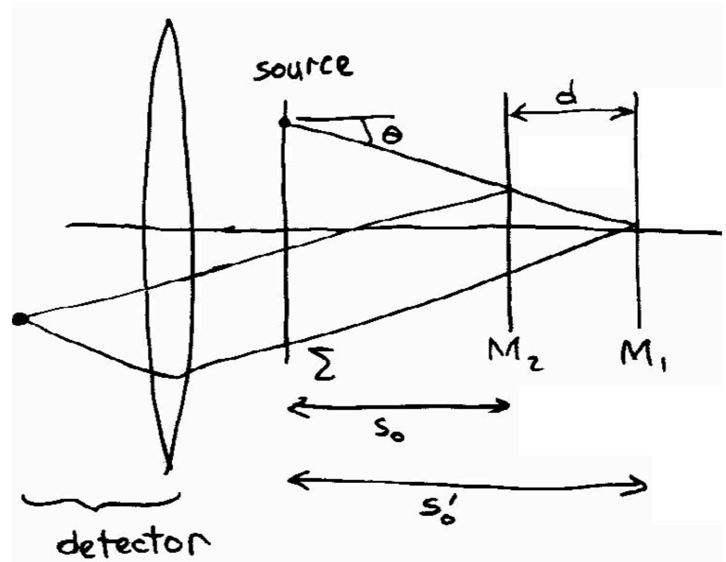
Michelson Interferometer:



M1, M2 – mirrors; one is movable

C – compensator (matches glass of BS)

equivalent view:



OPD $\Lambda = 2d\cos\theta$

phase diff $\delta = k_o\Lambda + \delta_R$

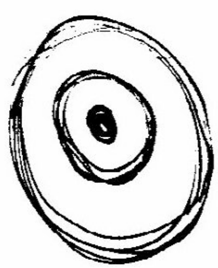
reflections; say BS is uncoated glass plate $\delta_R = \pm\pi$

$$= \frac{2\pi}{\lambda_o} 2d\cos\theta + \pi$$

minima at $\delta = (2m+1)\pi$

$$\Rightarrow \frac{1}{\lambda_o} 2d\cos\theta = m \quad , \quad 2d\cos\theta_m = m\lambda_o$$

M1, M2 aligned, see circular fringes:

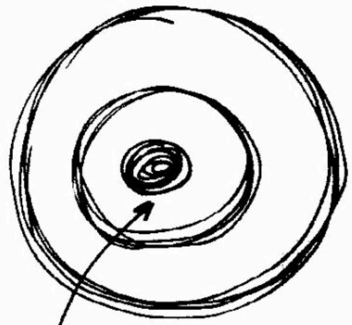


Haidinger fringes

M1, M2 tilted rel. to each other, see straight fringes:

Fizeau fringes

circular fringes:



central dark fringe

some $m = m_0$, $\theta_m = 0$

$$\Rightarrow 2d = m_0 \lambda_0$$

for fixed d, successive dark rings satisfy:

$$2d\cos\theta_1 = (m_0 - 1)\lambda_0$$

$$2d\cos\theta_2 = (m_0 - 2)\lambda_0$$

$$\vdots$$

$$2d\cos\theta_p = (m_0 - p)\lambda_0$$

or $\quad 2d\cos\theta_p = 2d - p\lambda_0$

$$2d(1-\cos\theta_p) = +p\lambda_0$$

using $1-\cos\theta_p \simeq +\dfrac{\theta_p^2}{2}$

$$\Rightarrow \theta_p \approx \left(\frac{p\lambda_0}{d}\right)^{1/2}.$$

thus, see

when far from $d=0$, and

when near $d=0$.

Also, if consider emergence (or disappearance) of fringes at fixed θ, say $\theta=0$, then

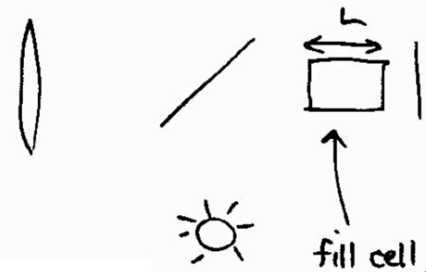
$$2d = m\lambda_0$$

$$\text{or} \quad 2\Delta d = \Delta m \lambda_0$$

with known λ_0, count Δm and calibrate Δd (distance scale).

applications:

i) index of refrac. of gas



fill cell, count fringes

$$\Delta d = nL - L$$

$$= (n-1)L = \frac{\Delta m}{2}\lambda_0 \qquad \text{(get n)}$$

ii) wavelengths of closely spaced lines

- when fringes coincide $m\lambda_0 = m'\lambda_0'$
- change Δd until they coincide again,

say initial $m = m' + N$

$$\frac{2d_1}{\lambda_0} = \frac{2d_1}{\lambda_0'} + N$$

next coincidence,

$$\tilde{m} = \tilde{m}' + (N+1)$$

$$\frac{2d_2}{\lambda_0} = \frac{2d_2}{\lambda_0'} + (N+1)$$

subtract earlier eqn: $\Delta d \equiv d_2 - d_1$

$$2\Delta d\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_0'}\right) = 1 \Rightarrow \Delta\lambda = \frac{\lambda_0\lambda_0'}{2\Delta d} = \frac{\lambda_{AV}^2}{2\Delta d}$$

Another example of amplitude splitting:

$n_2 > n_f > n_1$ (eg flouride film on glass, in air)

$$\Rightarrow \delta_B - \delta_A = 0.$$

for latter case make <u>anti-reflection coating</u>:

min in reflec for

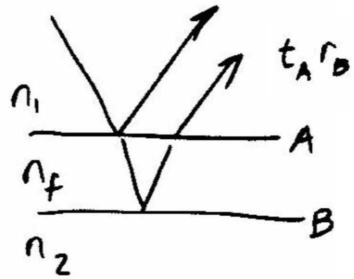
$$d\cos\theta_t = \frac{\lambda_f}{4}$$

or, for normal incidence, $d = \frac{\lambda_f}{4}$

(with white light source, see color).

- considering only 2 beams here, i.e. r_A and r_B small $\left(r = \pm\frac{n_i - n_t}{n_i + n_t}\right) \Rightarrow \Delta n$ small
- complete destructive int. for equal intensities of reflected beams, solve for n's to give equal I:

$$r_A = \pm \frac{n_1 - n_f}{n_1 + n_f} = \pm \frac{\Delta n_{1f}}{2n_1 - \Delta n_{1f}} \approx \pm \frac{\Delta n_{1f}}{2n_1}\left(1 + \frac{\Delta n_{1f}}{2n_1}\right)$$



$$t_A r_B t_A' = \frac{2n_f}{n_f + n_1} (\pm) \frac{n_f - n_2}{n_f + n_2} \frac{2n_1}{n_f + n_1}$$

$$= \frac{2n_f}{2n_f - \Delta n_{f1}} (\pm) \frac{\Delta n_{f2}}{2n_f - \Delta n_{f2}} \frac{2n_1}{2n_1 - \Delta n_{1f}}$$

$$\approx \frac{2n_f}{2n_f}\left(1 + \frac{\Delta n_{f1}}{2n_f}\right) (\pm) \frac{\Delta n_{f2}}{2n_f}\left(1 + \frac{\Delta n_{f2}}{2n_f}\right) \times \frac{2n_1}{2n_1}\left(1 + \frac{\Delta n_{1f}}{2n_1}\right)$$

want $r_A = t_A r_B t_A'$

$$\Rightarrow \frac{\Delta n_{1f}}{2n_1} = \frac{\Delta n_{f2}}{2n_f}$$

$$\frac{n_1 - n_f}{2n_1} = \frac{n_f - n_2}{2n_f} \quad , \quad \frac{1}{2} - \frac{n_f}{2n_1} = \frac{1}{2} - \frac{n_2}{2n_f}$$

$$\Rightarrow \frac{n_f}{n_1} = \frac{n_2}{n_f} \quad \text{or} \quad n_f = \underline{\underline{\sqrt{n_1 n_2}}}$$

eg. glass, air $n = \sqrt{1.5 \cdot 1} = 1.22$

use MgF_2 , $n = 1.38$